AP Statistics
Test: Inference for Proportions
Instructor's Version

Free Response: 2 questions, 52 points, 30 minutes

1. George and Jerry are working on their final project for AP Statistics. They wish to investigate the question, "Do people really read documents they are asked to sign?" They believe that fewer than half of all people read a document they have been asked to sign. To test their theory, they randomly select 50 individuals from various locations and then present them with various fake documents. After being presented with the documents, the individuals are asked to sign them. George and Jerry then observe whether or not the individuals read the document. Out of the 50 individuals they selected, 7 read the document then refuse to sign; 15 read it and sign; and the remaining 28 do not read the document, but sign it anyway. Does the sample result provide sufficient evidence to support George and Jerry's theory that fewer than half of all people actually read a document they have been asked to sign?

A. Carry out an appropriate test at a 5% level of significance to test George and Jerry's theory. (22 points)

Answer:

Identify parameter of interest: p = the true proportion of individuals who will read a document they are asked to sign

Hypotheses: $H_o : p = 0.5$ $H_\alpha : p < 0.5$

Name of test: *z*-test for one proportion

Conditions:
1. Sample is random — stated in problem
2. Normality — check to see if the expected number of successes and failures is at least 10. If so, we are allowed to use the *z*-test. Check that $np_o \geq 10$ and that $n(1-p_o) \geq 10$.
   50(0.5) = 25 and 50(0.5) = 25.
   This ensures that the sample is large enough to use Normal procedures.
3. Independent observations — this is reasonable since it is likely that the number of people who are asked to read and sign a document should be at least 10(50) or 500($N \geq 10n$)

Mechanics: By calculator: z = -0.8485

By hand: $z = \dfrac{0.44 - 0.5}{\sqrt{\dfrac{0.5(0.5)}{50}}} = \dfrac{-0.06}{0.0707} = -0.8487$

p-value = 0.1981
normalcdf(-0.8485,E99)=0.1981.
Decision Fail to reject H_o because *p*-value, 0.1981 > 0.05, sig. level

Conclusion: The sample does not provide sufficient evidence to support George and Jerry's theory that less than half of all people read a document they are asked to sign.

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Scoring notes:
Award 2 points for correctly indentifying the parameter of interest.
Award 2 points for a correct set of hypotheses.
Award 2 points for correct test name. If the student shows the calculation for the test statistic, *z*, then he will receive these 2 points even if he has not specifically stated the test name.
Award 2 points for each correct condition (for a maximum of 6 points).
Award 2 points for each correct test statistic, *z* and *p*-value (for a maximum of 4 points). Do not subtract points if the student calculates by hand and makes an arithmetic error in calculating *z*.
Award 2 points for correct decision with reason. Award only 1 point if no reason is given.
Award 4 points for a correct conclusion with context. Do not award these points if there is no context.

B. Interpret the *p*-value you calculated in part A. (6 points)

Answer:
Assuming the null hypothesis is true, there is about a 19.8% chance of obtaining a sample result as extreme or more extreme than the one obtained, $\hat{p} = 0.44$.

Scoring notes:
Award 6 points for a correct interpretation of the *p*-value. If wrong *p*-value is given in part A, but student gives a correct interpretation, award full credit.

C. Construct and interpret a 95% confidence interval for *p*. (8 points)

Answer:
95% CI is (0.30241, 0.57759)

0.44+-1.96(square root(((0.44)(0.56))/50)=0.44+-1.96*0.0702=0.44+-0.137592 = (0.30241,0.57759)

Conditions check: Verify that $n\hat{p} \geq 10$ and $n(1-\hat{p}) \geq 10$
$50(0.44) = 22$ and $50(0.56) = 28$

Interpretation: It can be estimated with 95% confidence that the true population proportion of people who read a document they are asked to sign is between 0.30241 and 0.57759.

Scoring notes:
Award 2 points for correct confidence interval.
Award 2 points for conditions check.
Award 4 points for a correct interpretation in context. Do not award credit without context.

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2. Say that you wish to construct a 90% confidence interval for the difference of two proportions, $p_1 - p_2$.

A. What conditions must be checked in order for you to perform the calculation and be sure that it will be reasonably accurate? (8 points)

Answer:
1. Both samples must be randomly selected.
2. Sample size should be sufficiently large, which is verified by checking that $n_1\hat{p}_1 \geq 5$, $n_1(1-\hat{p}_1) \geq 5$, $n_2\hat{p}_2 \geq 5$, and $n_2(1-\hat{p}_2) \geq 5$.
3. Independent samples: Check by verifying that both populations are at least 10 times the size of their respective samples.

Scoring notes:
Award 2 points each for conditions 1 and 3 (for a maximum of 4 points).
Award 2 points for condition 2 if all four calculations are shown.
Award 1 point for each correct calculation shown (for a maximum of 4 points).

B. Assuming all conditions are met, calculate a 90% confidence interval for $p_1 - p_2$ where $\hat{p}_1 = 0.816$, $\hat{p}_2 = 0.630$, and where *m* (margin of error) = 0.132. (4 points)

Answer:
$(\hat{p}_1 - \hat{p}_2) \pm m = (0.816 - 0.630) \pm 0.132 = 0.186 \pm 0.132 = (0.054,\ 0.318)$

Scoring notes:
Award 4 points for a correctly calculated confidence interval.

C. A significance test is run on the difference of the two proportions, p_1 and p_2, with hypotheses as follows:

$H_o : p_1 - p_2 = 0$
$H_\alpha : p_1 - p_2 \neq 0$

Based on the 90% CI calculated in part B, what decision would you expect to make regarding the null hypothesis when testing at $\alpha = 0.10$ and why? (4 points)

Answer:
Reject the null hypothesis since zero is not included in the CI.

Scoring notes:
Award 4 points for a correct decision regarding null hypothesis. If the student's calculated CI includes zero through arithmetic error, the decision should be to fail to reject the null hypothesis since zero would be included in the CI.

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